Schaefer, Brad E.: New methods and techniques for historical astronomy and archaeoastronomy. In: Archaeoastronomy: The journal of astronomy in culture XV (2000), pp. 121-135.

The scribbles in the margins are from V.M.M. Reijs. There are some typos on pages 128 and 129. Rectification of these can be seen below (or on:

http://www.archaeocosmology.org/eng/archxv.htm)

Typos and additions to Arch XV text

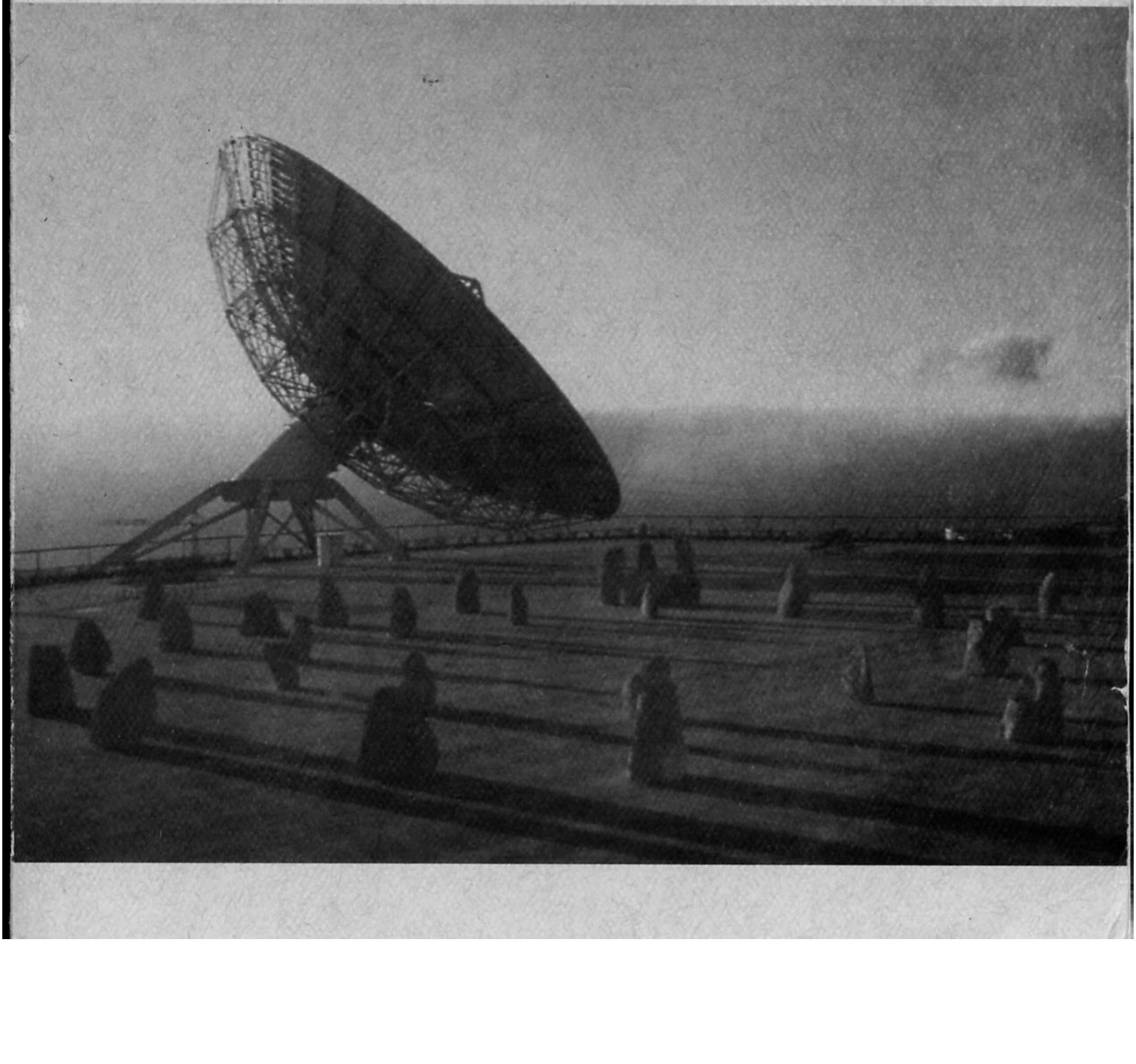
The following **typos** and dimension additions in Schaefer [2000] are found [pers. comm. Schaefer [2001]):

- H = site height (altitude) [m]
- λ =wavelength [μm]
- S = relative humidity [factor between 0 - 1]
- ϕ = site's latitude [rad]
- α_s = RA of sun [°]
- $k_{oz}=M_{oz,\lambda}$ * {1+0.13(ϕ*cos[α_s]-cos[3**ϕ**])}
- Z=zenith distance [°]
- H_L=height of extinction layer [km]
- $X_L(H_L)$=(1-[sin{Z}/{1+(H_L/R_o)}]**^{2}**)$^{-0.5}$
- B=surface brightness in **[erg/cm^2/sec/μm/arcsec^2]**
- α=phase angle of Moon in [°]
- h_{sun}=altitude of Sun above horizon in [°]
- ρ_{moon}=angular distance to Moon [°]
- ρ_{sun}=angular distance to Sun [°]
- B_{sky} in [nL]=**B_{sky}/1.02*10^{-15}**
  The value is different then Bradley uses in his BASIC program, see my discussion on this.
- ζ=angular diameter of extended source [']
- S**N**=Snellen ratio of observer
- θ_{CVA}=(40"/S**N**)*(10^8.28*B$^{-0.29}$)] if Log(B)<3.17
- θ_{CVA}=min[900",(380"/S**N**)*(10^**0.3***B$^{-0.29}$)] if Log(B)>**=**3.17
- log(C_{th})=-[0.12+0.4*log(B)]+[0.9-0.15*log(B)**]***log(100'/ζ) if log(B)<6 **and** ζ>20'

ARCHAEOASTRONOMY

THE JOURNAL OF ASTRONOMY IN CULTURE

VOLUME XV 2000



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Astronomy and Cultural Diversity: Oxford VI and SEAC99 Proceedings, La Laguna, Canary Islands

FRONT COVER PHOTO: Modern and ancient astronomy at the main terrace of the Museo de la Ciencia y del Cosmos in La Laguna, Tenerife, Canary Islands, Spain. Photograph by Museo de la Ciencia y del Cosmos

BACK COVER PHOTO: Themis Solar Telescope at Observatorio del Teide in Tenerife, Canary Islands, Spain. Photograph by The-mis S. L.

CORRECTION:
Please note that *Archaeoastronomy* 14:2 omitted credit to Clive Ruggles for the front cover and back cover photos.

New Methods and Techniques for Historical Astronomy and Archaeoastronomy

BRADLEY E. SCHAEFER

Abstract

Archaeoastronomy and history of astronomy are fields that are evolving fairly rapidly, and so are the techniques and methods employed by researchers. This review will cover new techniques and methods on three topics: celestial visibility, three-dimensional analysis software, and statistical analysis. All require computations for which the old or traditional tools are inadequate. By the nature of the topics, many of the most useful details are highly specific (such as lists of websites) or very technical (model equations). I have provided specific references for both theory and observation, references and websites for computer programs, graphs of typical results, explicit complete sets of equations for models, and references to exemplars of the new techniques.

Celestial Visibility

What is, and is not, visible in the sky is often the crux of many problems in archaeoastronomy and the history of astronomy. In general, the answer will require a detailed calculation. Examples include visibility of the thin crescent for calendric purposes, the azimuth of first visibility of stars for alignments, the date of heliacal rise events for comparison with lore, and the variation of refraction near the horizon for the reproducibility of solar alignments.

In the past, a variety of rules of thumb have been developed to quantify limits on celestial visibility. Unfortunately, all of these rules are not up to modern standards of accuracy. Indeed, in many cases, the errors caused by using the rules of thumb are substantial enough to yield wrong conclusions. This section will present resources (references for computer programs, research papers, and detailed model equations) and typical results (as graphs or equations) that will allow most researchers to access the new results.

This section will also address a series of questions about celestial visibility of increasing complexity, each discussed in a subsection.

Bradley E. Shaefer has been applying the techniques of celestial visibility, what is and is not visible in the sky, to a wide variety of historical problems. Recent work has looked at the early history of the Greek constellations; typically using precession to get the latitude and epoch for the observer/creator of the Almagest star catalog, the southern constellations, and the astronomical lore in Aratus as well as in Hesiod.

A bold-faced header denotes the beginning of each subsection. The final seven subsections will be lists of equations for use in constructing physical models.

Visual Limiting Magnitude

How faint an object can the human eye see at night? That is, what is the limiting visual magnitude? The traditional answer (dating from the Greeks) is that a sixth-magnitude star is at the visual threshold. However, determining the real visual limits is actually a fairly complex question both from an observational and a theoretical point of view. The limits depend on the proximity and phase of the Moon, the haziness of the air, the acuity of the observer, and the altitude of the star. As these conditions change over normal ranges, the limit will vary by up to six magnitudes.

All of these conditions can be quantified and incorporated into a physical/physiological model. The physical model calculates the extinction coefficient of the air (from the relative humidity, the altitude above sea level, the time of year, the latitude, and the ground temperature), the optical path length through the atmosphere (as a function of the altitude of the star for each of the four extinction components; Rayleigh scattering off gas molecules, Mie scattering off aerosols, selective extinction from ozone, and selective extinction from water vapor), and the sky brightness (as a function of the altitudes, distances, phase of the Moon, and position of the Sun). These combine to yield the perceived brightness of the star and the background. Then, physiological equations can indicate whether the star would be visible under these conditions. This visibility is a function of the observer's acuity as quantified by the Snellen ratio (20/20 for normal vision, 20/10 for the very best vision), which effectively gives the eye's pixel size. In general, specific and plausible values can be adopted for each input parameter, with the variation of these inputs yielding a variation in the output that will characterize the accuracy of the calculation.

Such physical/physiological equations can be used to construct models for all steps of the light transmission and detection process. In essence, the construction of these physical/physiological models is just treating celestial visibility as any normal problem in astrophysics. Once a model is created, then normal good science procedures demand that actual data be gathered to test the validity of the model predictions. Throughout this section, I will construct models of the many steps leading to the detection of the light, and then point to large observational databases against which the model was tested. These detailed tests are what then give confidence in the validity and accuracy of each model.

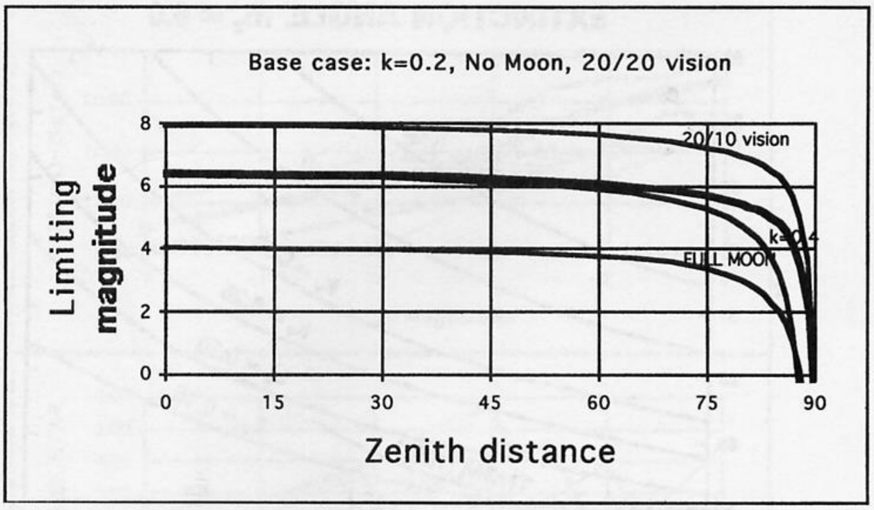


FIGURE 1. Nighttime limiting magnitude. The faintest visible star depends on the Moon, the observer's acuity, the atmospheric haziness, and the altitude above the horizon. A base case is plotted as a thick line. Thin lines are for cases that differ from the base case in only one input parameter, as labeled.

For the visibility of stars, a computer program is given in Schaefer (1999) and at http://www.skypub.com/resources/software/basic/programs/vislimit.bas. For detail relating to the theory, see Schaefer (1993a, 1993b). The relevant model equations are summarized at the end of this section. Several typical cases are displayed in the figure.

Extinction Angle

What is the altitude above the horizon at which a star will first appear? This has relevance for calculating the azimuth of first appearance of a star proposed for any alignment. Historically, Thom has proposed the rule of thumb that a star's altitude at first visibility (in degrees) will equal its visual magnitude. As we shall see below, such a rule corresponds to the extinction coefficient of the best high-altitude sites in the world. So when Thom's criterion is applied to ordinary sites at temperate latitudes, the deduced azimuth of first visibility is systematically in error by several degrees.

A satisfactory model can easily be constructed as a special case of the model in the previous subsection. For a computer program, see Schaefer (1987a) or http://www.skypub.com/resources/software/basic/programs/extinc.bas. For the full theoretical model, see Schaefer (1986, 1993a, 1993b). Schaefer (1986) contains reports on a series of observations used to test (and verify) the model predictions. Figure 2 displays typical results of the model for an average person (with 20/20 vision). ← not true!

For bright stars, the only important input to derive the extinction angle is the magnitude of the star and the extinction coefficient. It

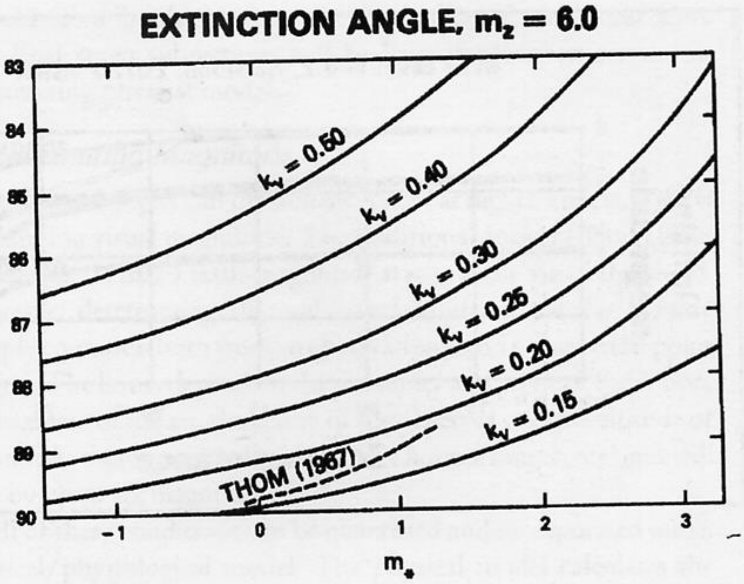


FIGURE 2. Extinction angle. How high above the horizon must a star rise before it is first visible at night? This depends on the magnitude of the star, the brightness of the sky, and the haziness of the atmosphere. The plot shows the zenith distance at the time of first visibility as a function of the star's magnitude for various extinctions under a normal dark sky.

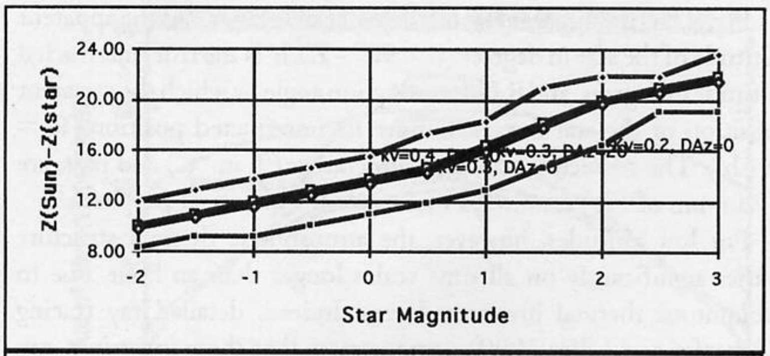
0.201 140 km purple
0.457 14 km green
40 km ~~red~~
0.27 50 km red
0.25 30 km
0.7

turns out that the acuity of the individual person is not important because the acuity enters in only as a product, while the extinction coefficient comes in as an exponential ($10^{-kX/2.5}$) which totally dominates the uncertainties at low altitude. This is a general property of low-altitude visibility.

Heliacal Rise

As the Sun moves through the sky, there is generally a period of time when an individual star is hidden by the Sun. On some date, the star will be just far enough from the Sun to be briefly visible in the dawn sky for the first time that season. This is called the date of heliacal rising, with the last date of a star's visibility in the evening twilight being termed the heliacal setting. Many old cultures have based their calendars on the heliacal rising and setting of stars, planets, and constellations. Examples include Sirius for the Egyptians, Canopus for the Arabs, Venus for the Maya, the Pleiades for the Aztec, and various constellations for the ancient Greeks. As such, the quantitative evaluation of heliacal rise dates is central to understanding the calendars and to correlating chronologies with the Julian calendar.

Until recently, the last set of published observations had been Babylonian data, while Ptolemy had made the last theoretical advance. Typical of rules of thumb, the results are only approximately valid for only one set of observing conditions, which are unknown. Fortunately, with a realistic model of twilight sky brightnesses, a detailed physical/physiological model can be constructed that is applicable to all observing conditions. As for all low-horizon



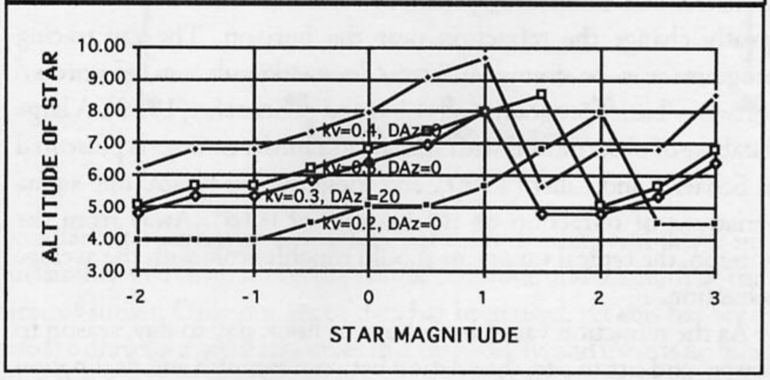


FIGURE 3. (TOP) *Arcus visionis* of stars. The *arcus visionis* is defined as the difference in zenith distance of the star and Sun at the instant of first visibility. The small filled diamonds are for the case with $k = 0.4$ and an azimuth difference of 0°. The empty squares are for the case with $k = 0.3$ and an azimuth difference of 0°. The empty diamonds are for the case with $k = 0.3$ and the Sun and star being 20° different in azimuth. The small filled squares are for the case of $k = 0.2$ and the star having the same azimuth as the Sun.

FIGURE 4. (BOTTOM) Altitude of stars at time of heliacal rising. Does a monument point in the direction of the heliacal rise of a star? The equation is easy, the observer's latitude known, and the star's declination known. The only tricky item is knowing the altitude of the star at the instant of first visibility. The plot gives this altitude for the same four cases as in Figure 3. Typical uncertainties in the extinction coefficient will translate into a several-degree uncertainty in the altitude of first visibility.

visibility questions, the uncertainties are entirely dominated by variations in the aerosol extinction coefficient.

A specific computer program for the heliacal rise model is in Schaefer (1985) or at http://www.skypub.com/resources/software/basic/programs/heliac.bas. For theory and a summary of a large database collected to test the model, see Schaefer (1987b). The two figures present model results on the *arcus visionis* (the difference in altitude between the star and Sun) and the altitude of the star alone as a function of the star's magnitude. Four cases are plotted, for the extinction coefficient and Sun-star azimuth differences of 0.4 and 0°; 0.3 and 0°; 0.2 and 0°; and 0.3 and 20°.

Refraction near the Horizon

The standard archaeoastronomy paradigm relates an azimuth to a declination, and this relation must account for the ordinary refraction of the atmosphere. Far away from the horizon or for average conditions (as for the U.S. Standard Atmosphere), the refraction will be given variously by

$R = 58.2'' * [(0.372*P)/(273° + T)] * \tan\{Z\}$ is accurate for $Z < \sim 80°$

$R = 1' / \tan\{h + (7.31°/[h + 4.4°])\}$ for all altitudes

$R = 1' / \tan\{h' + (10.3°/[h' + 5.11°])\}$ for all altitudes

Here, Z is the apparent zenith distance of the star, h is the apparent altitude of the star in degrees ($h = 90° - Z$), h' is the true unrefracted altitude in degrees, and R is the refraction angle by which the apparent position of the star is raised above its unrefracted position ($R = h - h'$). The corrections for air temperature (T in °C) and pressure (P in mm of Hg) are always minuscule for historical purposes.

For low altitudes, however, the atmospheric thermal structure varies significantly on all time scales longer than an hour, due to ubiquitous thermal inversion layers. Indeed, detailed ray tracing (Schaefer and Liller 1990) demonstrates that these inversions can greatly change the refraction near the horizon. The ray tracing programs can be found at http://www.skypub.com/resources/software/basic/programs/refr1.bas and in Schaefer (1989). A large database of observations with sextant and sunset timings is presented in Schaefer and Liller (1990), and these display typical one-sigma variations of refraction on the horizon of 0.18°. Away from the horizon, the typical variations should roughly scale with the average refraction.

As the refraction varies from hour to hour, day to day, season to season, and site to site, the relation between azimuth and declination will change also. These variations are essentially unpredictable and unknowable, so for a given site azimuth, the corresponding indicated declination will vary substantially. These refraction variations will set a lower limit on the possible accuracy of any archaeoastronomical alignment. For high horizons ($h > \sim 5°$) or equatorial sites, the variations in indicated declination will be small. Otherwise, the random variations will be significant. For the latitude of Scotland, low horizons will yield a full range of azimuth variations (for a given declination) comparable to a full degree. In essence, for many sites, refraction variation rules out the possibility of high-accuracy alignments.

Crescent Visibility

The visibility of the thin crescent moon in the evening sky is used by many cultures for calendric purposes. But without reliable prediction and postdiction algorithms, a calendar based on crescents loses much utility. Even today, the Islamic people still follow an observational lunar calendar. Given that, I would claim that the prediction of crescent visibility is the single most important "nontrivial" astronomical problem. This has led to much interest over the millennia; however, all of the old prediction criteria have little utility (Doggett and Schaefer 1994; Schaefer 1996). Since the early 1900s, another

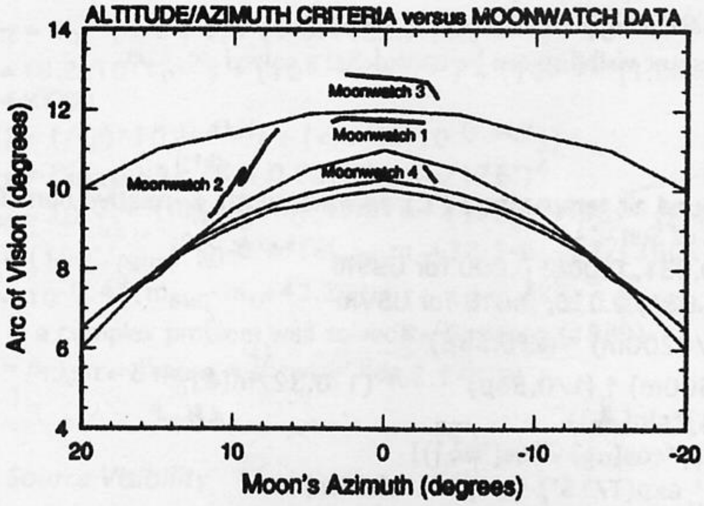


FIGURE 5. Altitude-azimuth criteria versus moonwatch data. The altitude of the Moon above the Sun (the arc of vision) must exceed some threshold value (as a function of the relative azimuth of the Sun and Moon) for the crescent to be sighted. Historically, the thresholds have all been derived from one set of data, yet even so, various investigators derive widely disparate results. In any case, the well-observed moonwatches show that the actual thresholds are widely variable from site to site and month to month. The reason for the failing of all altitude-azimuth criteria is that they do not account for the large variations in the haziness of the air as a function of the seasons, latitude, elevation, and relative humidity.

popular criterion has been to draw some curve dividing visibility from invisibility in a diagram of the Moon's altitude and azimuth at the time of sunset. Only one set of data has been used, yet this has been used to draw four separate curves that vary widely, and there is no hint that these curves should change as atmospheric haziness changes. Worse yet, observations from many hundreds of observers have been combined to show that the real dividing curve varies greatly with conditions (see Fig. 5 above). A detailed review of thousands of observations demonstrates that these empirical ALT-AZ rules of thumb can rarely make a confident prediction (Doggett and Schaefer 1994; Schaefer 1996).

Alternatively, lunar crescent visibility can be treated as a special case of the heliacal rise problem. The only changes are that an accurate lunar ephemerides is needed, the lunar surface brightness (in two angular dimensions) as a function of the lunar phase is required, and the difficult problem of the visibility of an extended source of varying surface brightness has to be tackled. Schaefer (1988, 1993a) discusses the theory and the associated computer program is available from Western Research Corporation (Tucson). This physical/physiological model has been strongly tested and verified against a set of thousands of observations taken during six moonwatches and collected from the literature (Doggett and Schaefer 1994; Schaefer, Ahmad, and Doggett 1993; Schaefer 1996). The strong conclusion is that the empirical algorithms are poor at best, while the theoretical model typically leads to uncertainties of ~30° in longitude for the line separating visibility from invisibility. With most of the remaining uncertainty arising from unknowable variations in the aerosol extinction coefficient, there is little likelihood for significant improvements. As such, crescent visibility can be considered a solved problem.

Extinction Coefficients

H=site's altitude, T=ground air temperature (°C), λ=wavelength, S=relative humidity
ϕ=site's latitude, α_S=RA of Sun

1 **$M_{OZ,\lambda}$=0.000, 0.000, 0.031, 0.008, 0.000 for UBVRI**

2 **$M_{W,\lambda}$=0.074, 0.045, 0.031, 0.020, 0.015 for UBVRI**

3 **k_R = 0.1066 * exp(-H/8200m) * $(\lambda/0.55\mu)^{-4}$**

4 **k_a = $\boxed{0.10}$ * exp(-H/1500m) * $(\lambda/0.55\mu)^{-1.3}$ * $(1\text{-}0.32/\ln[S])^{4/3}$ ***
(1+0.33*sign[ϕ]*sin[α_S])

5 **k_{OZ} = $M_{OZ,\lambda}$ * {1+0.13(ϕ*cos[α_S] - cos[3α_S])}**

6 **k_W = $M_{W,\lambda}$ * 0.94 * S * exp(T/15°) * exp(-H/8200m)**

(7) **k = k_R +k_a + k_{OZ} + k_W, σk = 0.01 +0.4*(k_a+k_W)**

Airmass

Z=zenith distance, H_e=scale height of extinction in km, H_L=height of extinction layer

8a **X = sec(Z) for low accuracy or for Z<~70°**

8b **X = (cos[Z] + 0.025 * exp[-11*cos{Z}])$^{-1}$ for reasonable accuracy**

9 **$X_e(H_e)$ = (cos[Z] + 0.01*$H_e^{0.5}$*exp[-30*$H_e^{-0.5}$*cos{Z}])$^{-1}$ for exponential extinction distribution**

10 **$X_L(H_L)$ = (1 - [sin{Z}/{1+(H_L/R_O)}])$^{-0.5}$ for extinction in layer, R_O=6378km**

Light Loss

I_{obs}=source intensity in foot-candles, I_{na}=source intensity with no atmosphere in ft-cd

11 **V = Johnson V magnitude = -16.57 -2.5*log(I_{na})**

12a **Δm = kX to a reasonable approximation**

12b **Δm = k_R*X_e(8.2km) + k_a*X_e(1.5km) + k_{OZ}*X_L(20km) + k_W*X_e(8.2km)**

13 **I_{obs} = I_{na} * $10^{-0.4*\Delta m}$**

Sky Brightness

B=surface brightness in nanoLamberts, Y=year,
α=phase angle of Moon in ° (FM for α=0°), h_{sun}=altitude of Sun above horizon in °
ρ_{moon}=angular distance to Moon, ρ_{sun}=angular distance to Sun, °

14 **$B_{o,av}$= 8x10^{-14}, 7x10^{-14}, 1x10^{-13}, 1x10^{-13}, 3x10^{-13} erg/cm^2/s/μ/"^{2} for UBVRI**

15 **C_{moon}= 1.36, 0.91, 0.00, -0.76, -1.17 mag for UBVRI**

16 **m_O= -10.93, -10.45, -11.05, -11.90, -12.70 mag for UBVRI**

17 **m_{sun}= -25.96, -26.09, -26.74, -27.26, -27.55 mag for UBVRI**

18 **B_O = $B_{o,av}$ * {1 + 0.3*cos[2π(Y-1992)/11]}**

19 $B_{night} = B_o * [0.4 + 0.6*(1-0.96*\sin^2\{Z\})^{-0.5}] * 10^{-0.4kX}$

20 $f(\rho) = (6.2\times10^7*\rho^{-2}) + (10^{6.15-\rho/40°}) + (10^{5.36}*[1.06+\cos^2\{\rho\}])$,
$\langle f\rangle=440000$

21 $Q(\rho,X) = (f(\rho)*10^{-0.4kX}) + (\langle f\rangle*[1-10^{-0.4kX}])$

22 $m_{moon} = C_{moon} - 12.73 + 0.026|\alpha| + (\alpha/126°)^4$ X moon

23 $B_{moon} = 10^{-0.4*(m_{moon}-m_o+43.27)} * (1-10^{-0.4kX}) * Q(\rho_{moon},X_{moon})$

24 $B_{twi} = (100°/\rho_{sun})\, 10^{-0.4*(m_{sun}-m_o+32.5-h_{sun}-Z/[360k])} * (1-10^{-0.4kX})$

25 $B_{day} = 10^{-0.4*(m_{sun}-m_o+43.27)} * (1-10^{-0.4kX}) * Q(\rho_{sun},X_{sun})$ Sun

26 B_{city} is a complex problem well solved by Garstang (1989) = 0

27 $B_{sky} = B_{night} + B_{moon} + \{B_{twi}$ or $B_{day}\} + B_{city}$

4? ?min

Point Source Visibility

28a $\log(C)=-8.35$ and $\log(K)=-5.90$ if $\log B_{sky}>3.17$

28b $\log(C)=-9.80$ and $\log(K)=-1.90$ if $\log B_{sky}<3.17$ ← B back ??

29 $I_{th} = C*(1+(KB_{sky})^{0.5})^2$

This threshold is for experienced observers, average binocular vision, natural fixation, and long observing times. The source will be visible if $I_{obs}>I_{th}$

Glare

Θ=angle to direction of glare source in degrees, D=telescope or eye diameter in cm

$B_{atm} = 6.25\times10^7*I_{na}*\Theta^{-2}*(10^{-0.4kX}-10^{-0.8kX})$

$B_{dif} = 1.13\times10^6*I_{na}*\Theta^{-3}*D^{-1}*10^{-0.4kX}$

$B_{mir} = 2.60\times10^8*I_{na}*\exp(-[\Theta/0.4]^2)$

$B_{eye} = 4.63\times10^7*I_{na}*\Theta^{-2}*10^{-0.4kX}$

$B_{glare} = B_{atm} + B_{dif} + B_{mir} + B_{eye}$, $B_{bck} = B_{sky} + B_{glare}$

Extended Source Visibility

S=Snellen ratio of observer (e.g., 20/20 has S=1, 20/10 has S=2)

ζ=angular diameter of extended source, D=telescope or eye aperture size in centimeters

B_{source}=surface brightness of source in nL, B=surface brightness of background in nL

$\Theta_{cva} = (40''/S)*(10\wedge[8.28*B^{-0.29}])$ if $\log(B)>3.17$

$\Theta_{cva} = \min[900'', (380''/S)*(10\wedge8.28*B^{-0.29})]$ if $\log(B)<3.17$ 0.3

$C = |B_{source} - B| / B$

$C_{th} = \max[2.4*B^{-0.1}, 20*B^{-0.4}]*(\Theta_{cva}/\zeta)^2$ if $\Theta_{cva}>\sim\zeta$

$C_{th} = 0.0028 + 2.4*B^{-0.1}*(\Theta_{cva}/\zeta)^2$ if $\log(B)>6$

$\log(C_{th}) = -[0.12+0.4*\log(B)]+[0.9-0.15*\log(B)]*\log(100'/\zeta)$ if $\log(B)<6$, $\zeta>20'$

This threshold is for experienced observers, average binocular vision, natural fixation, and long observing times

Three-Dimensional Analysis in Archaeoastronomy

Generally, in the past, analysis was two-dimensional. That is, site plans displayed the X-Y coordinates of stones, and planetarium programs displayed the ALT-AZ positions of celestial objects. In many real cases, however, the analysis problem is intrinsically three-dimensional, requiring an X-Y-Z coordinate system. Recently, workers have been introducing and using software that treats the problem in all three dimensions. This is a research trend that I applaud.

Tasks for which three-dimensional analyses are good include (1) the calculation of horizon profiles from topographic data, (2) the visibility of particular prominent mountains, (3) the intervisibility of archaeological sites, (4) the indicated declinations of alignments in a mountainous terrain, (5) delineation of shadows, and (6) educational/instructional tours of a site. Such programs can in part replace *in situ* observations, although the accuracy of any program must be confirmed by ground truth data. Once confidence is gained in a three-dimensional model, however, calculations can be used to evaluate shadows and alignments for conditions for which there is no practical chance of obtaining direct observations. Thus, models allow for discovering things year-round (without a resident monitor), through any cloud conditions (to avoid expense and frustration due to the weather), at different epochs (with the ancient obliquity not possible to determine today), and for likely shifts in the stone positions (which cannot be replaced in modern times). For some restricted classes of problems, the recent availability of three-dimensional software has allowed for great advances in visualization.

There is no one best three-dimensional software package for archaeoastronomy. In general, you will have to write your own, use some commercial package that is intended for some other purpose, or perhaps even combine more than one such program.

Global Information Systems (GIS) are managed databases that contain various types of data (e.g., height, vegetation, monument location) stored in a basically two-dimensional array to allow for the calculation of three-dimensional quantities (visibility of mountains, horizon profiles, intervisibility of monuments, alignments). General tutorials can be found at http://info.er.usgs.gov/research/gis/title.html and at http://www.census.gov/geo/www/faq-index.html.

A modern three-dimensional visualization standardized environment is called VRML. In theory, this is a very powerful and broad environment that has already spawned many applications from many companies. The problem is that it will take a significant effort to learn to use the programs and you will have to obtain the software. All of these programs need something like 32 Mb of RAM, 10 Mb of disk space, a fast graphics card, and usually operate only from Windows®. Victor Reijs recommends the following commercial programs:

- Notepad, a text editor to write VRML at http://www.vrml.org/
- Spazz3D, to make simple VRML models at http://www.spazz3d.com/
- SkyPaint, for sky backdrops and sun paths at http://www.wasabisoft.com/
- Genesis, realistic landscapes from topographic maps at http://www.geomantics.com/
- VRML browsers and plug-ins available at http://www.web3d.org/vrml/browpi.htm
- PovRay, ray tracing to make 3-D worlds at http://www.povray.org/ for many operating systems
- vrml2pov, VRML conversion to PovRay at http://grserv.med.jhmi.edu/~paul/vrml2pov/

The commercial software market is fast changing, so these packages will likely be outdated in a few years.

The software task of creating a three-dimensional model is too long and involved to be able to present in this review. Perhaps, however, an equally valuable service is to present a variety of cases where three-dimensional problems are central.

1. Victor Reijs has several impressive models on the web. One example is a model showing the play of light and shadow inside the tomb at Maes Howe in the Orkneys. On the winter solstice, the rising Sun casts light down the length of the tomb to light up the interior. Reijs has generated a three-dimensional model of the positions of the rocks and then calculates where and when light falls in the interior. Admirably, he has also checked his predictions by getting videotapes of the sunbeams in recent years. With a confident model, he can then analyze the play of light for any date and any time and any epoch. He can also examine the effects of hypothetical shifts in the positions of stones. His model is available at http://maeshowe.mypage.org/.
2. Another three-dimensional model by Victor Reijs is of the Treasury of Atreus in Greece. This is available at http://www.geniet.demon.nl/eng/aarde.htm. His web page also has an animation of the motion and shape of the triangle of light on the side of the *tholos* at the time of the equinox.
3. Two recent and good trends in archaeoastronomy have been the realization that alignments can be toward nonastronomical objects (like prominent mountains) and that the analysis of many sites can provide a confident conclusion that no one site by itself would allow. Perhaps the best exemplars of these trends are a series of three-dimensional GIS analyses of megalithic sites in Scotland (summarized in Ruggles 1999).

Typical tasks for the GIS software are to calculate the visibility of a particular peak along with its indicated declination for all points in the entire area of interest.

4. Stonehenge is the archetypal archaeoastronomical monument with broad public recognition. With tall trilithons, stones on the ground, and interesting horizon phenomena, Stonehenge offers a good opportunity for three-dimensional educational tours. Intel® offers one such tour as a demonstration of the power of its new computer chip, on-line at http://www.intel.com/cpc/explore/stonehenge/.
5. Stonehenge is surrounded by a large number of barrows. It is striking that from the monument itself virtually all barrows appear near the top of ridgelines. That is, barrows are not on the back sides of ridges nor do they appear in low spots on the near sides. This property of the barrow distribution (only on ridgelines from one central site) is presented from a GIS analysis by Cleal et al. (1995), where it is also shown that this property is only valid for a small area centered on the monument itself. The work by Cleal et al. also presents a sterling example of the use of Bayesian analysis in archaeology with respect to combining all the dating information for each of the building stages.

Statistical Analysis

Frequently, astronomy historians or archaeoastronomers will point to some set of coincidences or alignments as the primary evidence regarding the intentions of ancient peoples. In all such cases, the researcher must investigate and reject the null hypothesis. The null hypothesis is generally that the claimed coincidence or alignment is entirely by chance with no intention on the part of the builder/observer. Without a disproof of the null hypothesis or independent evidence in favor of the alignment/coincidence, Occam's Razor would select the null hypothesis as the simpler explanation. At a deeper level, prior expectations strongly favor the null hypothesis because all societies for which we have extensive knowledge have remarkably little astronomical content in everyday or even ritual conditions, so we would expect only rare and small astronomical content in lore and artifacts of ancient societies. Another fundamental reason for the requirement of testing the null hypothesis is that it serves as a representative alternative model. If more than one model of greatly different character fits the data, then no conclusion of any confidence can be reached. Thus, claims for which alignments or correlations are the primary evidence must be subjected to tests for whether the null hypothesis can be rejected.

Testing the null hypothesis is generally accomplished by a probabilistic or statistical analysis. Typically, some input parameter (e.g., an azimuth of a sight line, the number of marks on a wall) is presumed to be selected from some appropriate random distribution, and then the degree of correlation or alignment is compared to the actual data. If the random input reproduces the observed degree sufficiently rarely, then we would reject the null hypothesis.

For testing the null hypothesis relating to alignments, the basic (Bernoulli's) equation is

> $P = 1 - \Sigma\{[n! * p^{s} * (1 - p)^{n-s}] / [s! * (n-s)!]\}$, where P is the probability that at least "r" sight lines out of "n" independent sight lines with random azimuths will point close to preselected target azimuths. The summation in this equation is over "s" from 0 to $r - 1$. The probability p is the fraction of the horizon occupied by targets. For a simple case with the azimuthal accuracy of σ, $p = 8 * (2\sigma)/360°$ for the four cardinal and the four solstitial directions. If the lunar standstill points are included, then $p = 16 * (2\sigma)/360°$.

Probability calculations always have to be made with close attention to the question being asked, and in general every application is different. For questions other than those regarding alignments, other probability formulae will be needed. Likely, the best tutorial discussion of statistical questions in archaeoastronomy is in Ruggles (1999).

Threshold levels of > 3-sigma ($P < 0.0027$) are required in general. There are very fundamental and omnipresent reasons why even a 3-sigma result is to be viewed skeptically in almost all practical cases. First, there are always a large number of hidden trials, where a given researcher tries various possibilities until one is found that looks significant, and where many researchers try different sites or different tasks but only report the ones that appear significant. With enough hidden trials, there will always be an apparently significant result brought about by simple randomness. For example, if 10 researchers all look at different monuments in 10 different ways, then we expect to have one result that is significant at the $P = 0.01$ level. Since the negative results are rarely published and rapidly forgotten, the one published "success" will skew the field incorrectly. A second fundamental reason is that unknown systematic effects are important more often than we would like, and these will increase the uncertainties and lower the significance. A third fundamental reason common in archaeology is that the claimed pattern is recognized only after the data is in hand. The human mind is a tremendous pattern-recognition device (e.g., canals on Mars), and any complex data set has myriad possible patterns that could be published on. The bottom line, drawn from these three reasons, is the empirical truth that roughly half of all published 3-sigma results are false.

For archaeoastronomical alignments, the analysis from the last few paragraphs can be used to prove that no one site in itself can ever provide a significant alignment. That is, take the best possible case of a site with only one possible direction, say $\sigma = 1°$, and even ignore lunar alignments (so $p = 0.044$, $r = 1$, $n = 1$). In this best case, $P = 0.044$, which is not even a 2-sigma result, and the alignment is rejected as not being even near any civilized acceptance threshold. This shows a fundamental truth that no alignment at any one site can ever lead to a conclusion that is intentional without using outside information. In other words, an accurate site plan is never enough. Therefore, archaeoastronomers seeking alignments must get outside information. This outside information must come from archaeology, history, or ethnography, or through the use of multiple sites. Of necessity, archaeoastronomy must become interdisciplinary, since alignments alone can never convince.

Of the possible sources of outside information, the fourth is other sites of similar provenance. If many sites all show the same property, then a joint probability can readily become significant. (See Ruggles 1999:95 for a description of the probability computational issues.) For many of the most ancient sites (i.e., with no historical or ethnographic information), the use of multiple sites might become the only way to prove intent on the part of the builders. A. Thom was the first to statistically analyze many sites together, but there are few other examples until the last decade or so. Perhaps the most sterling examples have been the British megalithic surveys of C. Ruggles (summarized in Ruggles 1999) and the orientation of small monuments on islands and coasts around the Mediterranean (see a long series of articles in *Journal for the History of Astronomy* and *Archaeoastronomy* for the last decade, for example, Hoskin 1989; Hoskin and Palomo 1998). These extensive surveys should serve as exemplars for all other researchers interested in orientations of monuments with little historical or ethnographic data.

A recent trend in archaeology and archaeoastronomy is to apply Bayesian techniques as an alternative to the classical probability methods. Bayesian analysis is based on Bayes' theorem of conditional probability $P(A \mid B) = P(B \mid A)*P(A)/P(B)$, where A and B are data or hypotheses. The "$\mid$" symbol implies that the probability for the preceding item is stated on the condition that the following item is true. Reworking this into a test comparing two hypotheses,

$$P(H1 \mid Data)/P(H2 \mid Data) = \{P(Data \mid H1)/P(Data \mid H2)\}*\{P(H1)/P(H2)\}.$$

The left-hand side of the equation is the odds ratio between the two hypotheses H1 and H2 in light of all the data. The bracketed term just to the right of the equal sign is a statement about how likely the two hypotheses are to reproduce the observed data. The bracketed term on the far right is the ratio of the prior probabilities for the two hypotheses, that is, their comparative likelihood given all that you knew previous to the consideration of the new data. The middle term provides a quantitative estimate of how the new data should update your previous evaluation.

The Bayesian technique has definite advantages and disadvantages. Advantages are that it easily handles prior information, it readily can accommodate conditional measures, and it does not conceptually require repeated independent experiments. Disadvantages are that the prior knowledge is often hard to quantify and is subjective, while calculations are often difficult computationally. The weight of the pros and cons will vary greatly from problem to problem.

There has long been tension between the classical (or frequentist) and Bayesian schools of probability. Generally, the classical approach is taught in schools and widely used in science publications. But the Bayesians argue that natural human evaluations are essentially Bayesian and that most real situations do not allow for the multiple independent experiments required (at least conceptually) by the frequentists. The classical school generally tends to ignore the Bayesians, while Bayesian converts proselytize with a religious fervor.

So when you are approaching a new problem, should you choose to calculate probabilities with the classical or the Bayesian approach? Both the classical and Bayesian methods are perfectly correct. However, often they are used to address fundamentally different questions. By choosing one approach or the other, you will subtly change how you are using the data. Fortunately, I believe that similar relevant questions can be formulated in both approaches. I recommend that you base your choice on expedience. That is, perhaps it would take too long to learn Bayesian techniques, or perhaps the conditional nature of your problem makes for a simple calculation within the Bayesian paradigm, or perhaps the necessary Bayesian integrations are too complex to perform readily. Since both methods are valid, I suggest that you go with whichever path is fastest.

The "new" techniques in this section (the requirement for testing the null hypothesis, multisite analysis, and Bayesian procedures) are actually rather old. However, the common application to statistical analysis in archaeoastronomy has only started recently. All three trends are to be applauded as they are helping to make archaeoastronomy a confident science with rigorous conclusions.

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